ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: AUG -DEC-2022 **BATCH**: 2021 – 2025

BRANCH: ECE **YEAR/SEM:** II/03

SUB CODE/NAME: MA3355 –RANDOM PROCESSES AND LINEAR ALGEBRA

UNIT I – PROBABILITY AND RANDOM VARIABLES

PART – A

1. Define random variable.
2. X and Y are independent random variables with variances 2 and 3. Find the variance of 3X +4Y.
3. Let X be a R.V with E[X]=1 and E[X(X-1)]=4 . Find var X and Var(2-3X).
4. The number hardware failures of a computer system in a week of operations as the following pmf:

Number of failures:	0	1	2	3	4	5	6
Probability :	0.18	0.28	0.25	0.18	0.06	0.04	0.01

Find the mean of the number of failures in a week

5. A continuous random variable X has the probability density function given by $f(x) = 3x^2, 0 \le x \le 1$. Find K such that P(X > K)= 0.5
6. A random variable X has the pdf f(x) given by $f(x) = \begin{cases} Cxe^{-x}, x > 0 \\ 0, x \le 0 \end{cases}$. Find the value of C and c.d.f of X.
7. The cumulative distribution function of a random variable X is F(x)= $\left[1-(1+x)e^{-x}\right], x > 0$. Find the probability density function of X.
8. Is the function defined as follows a density function? $f(x) = \begin{cases} 0, x < 2 \\ \frac{1}{18}(3+2x), 2 \le x \le 4 \\ 0, x > 4 \end{cases}$
9. Let X be a R.V with p.d.f given by $f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, otherwise \end{cases}$. Find the pdf of Y =(3X +1).
10. Find the cdf of a RV is given by $F(x) = \begin{cases} 0, x < 0 \\ \frac{x^2}{16}, 0 \le x \le 4 \\ 1, 4 < x \end{cases}$ and find P(X>1/X<3).
11. A continuous random variable X that can assume any value between x = 2 and x = 5 has a density function given by f(x) = K(1 + x). Find P[X<4].
12. The first four moments of a distribution about x = 4 are 1, 4, 10 and 45 respectively. Show that the mean is 5, variance is 3, $\mu_3 = 0$ and $\mu_4 = 26$.
13. Define moment generating function.

14. Find the moment generating function for the distribution where $f(x)=\begin{cases}\frac{2}{3}, x=1\\ \frac{1}{3}, x=2\\ 0, otherwise\end{cases}$.

15. For a binomial distribution mean is 6 and S.D is $\sqrt{2}$. Find the first two terms of the distribution.

16. Find the moment generating function of binomial distribution.

17. The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution

18. If X is a Poisson variate such that P(X=2) = 9P(X=4) +90P(X=6),find the variance.

19. Write the MGF of geometric distribution.

20. One percent of jobs arriving at a computer system need to wait until weekends for scheduling , owing to core-size limitations. Find the probability that among a sample of 200 jobs there are no job that have to wait until weekends.

21. Show that for the uniform distribution $f(x)=\frac{1}{2a}, -a<x<a$ the moment generating function about origin is $\frac{\sinh at}{at}$.

22. If X is a Gaussian random variable with mean zero and variance σ^2, find the probability density function of $Y=|X|$.

23. A random variable X has p.d.f $f(x)=\begin{cases}e^{-x}, x>0\\ 0, x<0\end{cases}$. Find the density function of $\frac{1}{x}$

24. State Memoryless property of exponential distribution.

25. The mean and variance of binomial distribution are **5** and **4**. Determine the distribution.

26. For a binomial distribution mean is **6** and S.D is $\sqrt{\mathbf{2}}$. Find the first of the distribution.

27. What are the limitations of Poisson distribution.

28. A random variable X is uniformly distributed between 3 and 15.Find mean and variance.

29. A continuous random variable X has a p.d.f given by $f(x)=\begin{cases}\frac{3}{4}(2x-x^2), 0<x<2\\ 0 \quad , otherwise\end{cases}$ .Find $p(x>1)$

30. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of X.

31. If $f(x)=\begin{cases}ke^{-x}, x>0\\ 0, otherise\end{cases}$ is p.d.f of a random variable X ,then the value of K.

32. Find the mean and variance of the discrete random variable X with the p.m.f $p(x)=\begin{cases}\frac{1}{3}, x=0\\ \frac{2}{3}, x=2\end{cases}$

33. A random variable X has c.d.f $F(x)=\begin{cases}0 \quad , x<1\\ \frac{1}{2}(x-1), 1\le x\le 3\\ 0 \quad , x>3\end{cases}$. Find the p.d.f of X and the expected value of X.

PART – B

FIRST HALF (All are 8- marks)

(A) DISCRETE DISTRIBUTION:-

I- Binomial distribution

1. ***The moment generating function of a binomial distribution and find mean and variance (i.e)*** $p(x) = \begin{cases} nc_x p^x q^{n-x}, x = 0, 1, 2, 3 \ldots \ldots \\ 0 \qquad , otherwise \end{cases}$
2. The probability of a bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed?
3. If 10% of the screws produced by an automatic machine are defective find the probability that out of 20 screws selected at random, there are
   (i) exactly 2 defective (ii) at most 3 defective (iii) at least 2 defective

   (iv) between 1 & 3 defective
4. the probability of a man hitting a target is $\frac{1}{4}$. If he fires 7- times (i) what is the probability of his hitting the target at least twice? (ii) How many times must he fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$.
5. A machine manufacturing screws is known to probability 5% defective in a random sample of 15 screws, what is the probability that there are (i) exactly 3 defective (ii) not more than 3 defectives.
6. Five fair coins are flipped. If the outcomes are assumed independent find the probability mass function of the number of heads obtained.

II- Poisson distribution

1. ***The moment generating function of a Poisson distribution and find mean and variance (i.e)*** $P(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, 3, \ldots \ldots \ldots \\ 0 \qquad , otherwise \end{cases}$
2. ***Derive the Poisson distribution as a limiting case of binomial distribution***.
3. If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.

4. The no.of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8, Find the probability that this computer will function a month (i) without a breakdown (ii) with only one breakdown (iii) with at least one breakdown .

5. If X is a Poisson variate such that $p(x=1)=\frac{3}{10}$ and $p(x=2)=\frac{1}{5}$, find $p(x=0)$ and $p(x=3)$.

6. If X is a Poisson variate such that $P\,(X=2)=9\,P(X=4)+90\,P(X=6)$ find the mean and variance.

III- <u>Geometric distribution</u>

1. The moment generating function of a Geometric distribution and find mean and variance (i.e) $p(x)=q^{x-1}P$, $x=1,2,3,\ldots\ldots$ $where\ q=1-p$.

2. ***State and prove the memory less property of the geometric distribution***.

3. Suppose that a trainee soldier shoots a target in a independent fashion. If the probability that the target is shot on any one shot is 0.8. (i) what is the the probability that the target would be hit on 6^{th} attempt (ii) what is probability that it takes him less than 5 shots (iii) what is probability it takes him even no. of shots.

4. If the probability that a target is destroyed on any one shot is 0.5, what is the probability that it would be destroyed on 6^{th} attempt?

5. If the probability that an applicant for a driver license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the 4^{th} trial. (ii) in fewer than 4 trials.

<u>Problems on Discrete random variables:-</u>

1. If the probability mass function of a random variable X is given by $P[X=x]=kx^3, x=1,2,3,4,$ (i) find the value of K.(ii) $P\left[\left(\frac{1}{2}<X<\frac{5}{2}\right)/X>1\right]$
(ii) Mean and variance.

2. A random variable X has the following probability function

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find the value of K (ii) $P(X<6)$ (iii)$P(X\geq 6)$ (iv) $P(1\leq x\leq 5)$

V.PRAKASH,MS.c,M.Phil,B.Ed,
Associate Professor
Department Of Mathematics

SECOND HALF (All are 8- marks)

(B) Continuous DISTRIBUTION:-

I- Uniform distribution

1. The moment generating function of a uniform distribution and find mean and variance

2. Buses arrive at a specified bus stop at 15-min intervals starting at 7 a.m that is 7a.m ,7.15am ,7.30am,…..etc. If a passenger arrives at the bus stop at a random time which is uniformly distributed between 7am and 7.30 am.Find the probability that he waits (i) less than 5 min (ii) atleast 12 min for bus.

3. Subway trains on a certain run every half an hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20-minutes.

II- Exponential distribution

1. *The moment generating function of a exponential distribution and find mean and variance.* (i.e) $P(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0, \lambda > 0 \\ 0 \quad , otherwise \end{cases}$

2. *State and prove the memory less property of the exponential distribution.*

3. The time in hours required to repair a machine is exponentially distribution with perimeter $\lambda = \frac{1}{2}$. (i) what is probability that the repair time exceeds 2 hours. (ii)What is the conditional probability that repair takes at least 10 hours given that is duration exceeds 9 hours.

4. The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that the person will take for (i) more than 8-minutes (ii) between 4 and 8 minutes.

III- Normal distribution

1. The moment generating functions of a normal distribution and find mean and variance.
2. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance.
3. In a distribution exactly normal 75 of the items are under 35 and 39% are under 63. What are the mean and S.D of the distribution.
4. The peak temperature T, as measured in degrees Fahrenheit on a particularday is the Gaussian (85,10) random variables. What is $P(T > 100)$, $P(T < 60)$, $P(70 < T < 100)$.
5. An electrical firm manufactures light bulbs that have a life before burn out that is normally distributed with mean equal to 800 hrs and a standard deviation of 40 hrs. Find the (i) the probability that a bulb more than 834 hrs. (ii) the probability that bulbs burns between 778 and 834 hrs.

(C) Problems on continuous random variables:-

1. A continuous random variable X that can assume any value between X=2 and X=5 has a probability density function given by $f(x) = k(1+x)$. Find $P(X < 4)$.

2. A continuous random variable X has pdf $f(x) = \begin{cases} \frac{k}{1+x^2}, & -\infty < x < \infty \\ 0 & , otherwise \end{cases}$

Find (i) the value of K. (ii) $P(X > 0)$ (iii) distribution function of X.

3. If X is a continuous R.V's whose pdf is given by $f(x) = \begin{cases} c[4x - 2x^2], & 0 < x < 2 \\ 0 & , otherwise \end{cases}$

4. The probability distribution function of a R.V's is $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

Find the cumulative distribution function.

5. The density function of a R.V's X is given by $(x) = Kx(2-x), 0 \leq x \leq 2$.
Find the mean and variance.

6. The c.d.f of a R.V's X is $F(x) = 1 - (1+x), x \geq 0$. Find the p.d.f of X, mean and variance.

V.PRAKASH,MS.c,M.Phil,B.Ed,
Associate Professor
Department Of Mathematics

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UNIT-2 TWO-DIMENSIONAL RANDOM VARIABLES

PART – A

1. If two random variables X and Y have probability density function (PPF) $f(x,y) = ke^{-(2x+y)}$ for x ,y >0,evaluate k.
2. Define joint probability distribution of two random variables X and Y and state its properties.
3. If the point pdf of (X,Y) is given by $f(x,y) = e^{-(x+y)} x \geq 0, y \geq 0$ find E[XY].
4. If X and Y have joint pdf $f(x,y) = \begin{cases} x+y, 0<x<1, 0<y<1 \\ 0, otherwise \end{cases}$, check whether X and Y are ndependent.
5. Find the marginal density functions of X and Y if $f(x,y) = \frac{2}{5}(2x+5y), 0 \leq x \leq 1, 0 \leq y \leq 1$.
6. If the function $f(x,y) = c(1-x)(1-y), 0<x<1, 0<y<1, y<1$ to be a density function,find the value of c.
7. Let X and Y be continuous RVs with J.p.d.f $f(x,y) = \begin{cases} 2xy + \frac{3}{2}y^2, 0<x<1, 0<y<1 \\ 0, otherwise \end{cases}$ .Find P(X +Y<1).
8. The regression lines between twqo random variables X and Y is given by 3X +Y =10 and 3X +4Y =12. Find the co-efficient of correlation between X and Y.
9. If X and Y are random variables such that Y = aX +b where a and b are real constants, show that the correlation co-efficient r(X,Y) between that has magnitude one.
10. If Y = -2X +3 , find the cov(X,Y).
11. Let (X,Y) be a two dimensional random variable. Define covariance of (X,Y). If X and Y are independent , what will be the covariance of (X,Y).
12. The regression equations of X on Y and Y on X are respectively 5x –y =22 and 64x -45y =24. Find the means of X and Y.
13. The tamgent of the angle between the lines of regression Y on X and X on Y is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$. Find the correlation coefficient.
14. State the central limit theorem for independenc and identically distributed random variables.
15. The two regression equations of two random variables X and Y are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. find mean values of X and Y.

16. The angle between the two lines of regression.

17. If X and y are independent random variables with variance 2 and 3, then find the variance of $3x + 4y$.

18. The lines of regression in a bivariate distribution are $x + 9y = 7$ and $y + 4x = \frac{49}{3}$. find the coefficient of correlation.

19. If the joint P.d.f of (x, y) is $f(x, y) = \begin{cases} \frac{1}{4}, & 0 \le x, y \le 2 \\ 0, & \text{otherwise} \end{cases}$ Find $P[x + y \le 1]$

20. If two random variable X and Y have P.d.f $f(x, y) = Ke^{-(2x+y)}$, for $x, y \ge 0$. Find the value of K.

21. Find K if the joint p.d.f of a bivariate random variable is given by $f(x, y) = \begin{cases} k(1 - x)(1 - y), & 0 < (x, y) < 1 \\ 0, & \text{otherwise} \end{cases}$

22. 8. If the joint p.d.f of (x, y) is $f(x, y) = \begin{cases} e^{-(x+y)}, & x>0, y>0 \\ 0, & \text{otherwise} \end{cases}$ Check whether X and Y are independent.

23. The joint p.m.f of a two dimensional random variable (x,y) is given by $\mathbf{P(x, y) = K(2x + y)}$, $\mathbf{x = 1, 2}$ and $\mathbf{y = 1, 2}$, where K is a constant. Find the value of K.

PART – B

FIRST HALF (All are 8- marks)

I- Problems on discrete random variable

1. The joint PMF of two random variables X and Y is given by

   $P(x, y) = \begin{cases} k(2x + y), & x = 1,2; y = 1,2 \\ 0, & \text{otherwise} \end{cases}$, where K is a constant.

   (i) Find the K. (ii) Find the marginal PMF of X and Y.

2. The joint probability mass function of (x,y) is given by $P(x, y) = \frac{1}{72}(2x + 3y)$ $x = 0,1,2$ *and* $y = 1,2,3$. Find all the marginal and conditional probability distribution of X and Y.

3. The joint probability mass function of (x,y) is given by $P(x, y) = K(2x + 3y)$ $x = 0,1,2$; $y = 1,2,3$. (i) Find all the marginal and conditional probability distribution. Also find the probability distribution of $(x + y)$ *and* $P[x + y > 3]$

4. The joint distribution of X and Y is given by $f(x, y) = \frac{x+y}{21}$, $x = 1,2,3; y = 1,2$. Find the marginal distribution. Also find $E[xy]$.

5. Three balls are drawn at random without replacement from box containing 2- white , 3-red, 4-black balls. If X denote the no.of white balls and Y denote the no.of red balls drawn.Find the joint probability distribution of (X,Y)

II- *Problems on continuous random variable*

1. The joint PDF of (x,y) is $f(x,y) = e^{-(x+y)}, x,y \geq 0$. Are X and Y independent?
2. If the joint probability distribution function of a two dimensional random variable (x,y) is given by $f(x,y) = \begin{cases} (1-e^{x})(1-e^{y}), & x>0, y>0 \\ 0 & ,\text{otherwise} \end{cases}$. (i) Find the marginal densities of X and Y. Are X and Y independent? (ii) $P[1 < x < 3 , 1 < y < 2]$.
3. Given the joint density function of X and y as $f(x,y) = \begin{cases} \frac{1}{2} xe^{-y} ; 0 < x < 2 , y > 0 \\ 0 \quad ,elsewhere \end{cases}$. Find the distribution X+Y.
4. The joint PDF of the random variables (x,y) is given by $f(x,y) = k\, xy\, e^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of K and also Prove that x and Y are independent.
5. If X and Y are two random variable having joint density function $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), 0 < x < 2 , 2 < y < 4 \\ 0 \quad , \ otherwise \end{cases}$ .Find (i) $P[x < 1 \cap y < 3]$ (ii) $P[x+y < 3]$ (iii) $P[x < 1 / y < 3]$.
6. Suppose the point probability density function is given by $f(x,y) = \begin{cases} \frac{6}{5} (x+y^2); 0 < x < 1 , 0 < y < 1 \\ 0 \quad ,elsewhere \end{cases}$. Obtain the marginal P.d.f of X and Y. Hence find $P\left[\frac{1}{4} \leq y \leq \frac{1}{2}\right]$
7. Given $f_{xy}(x,y) = c\, x(x-y), 0 < x < 2; -x < y < x,$
   (i) Evaluate C (ii) Find $f_x(x)$ and $f_y(y)$ (iii) $f_{\frac{x}{y}}\left(\frac{x}{y}\right)$.

III- *Problems on correlation and covariance*

1. The joint PDF of a random variable (x,y) is $f(x,y) = 25e^{-5y}, 0 < x < 0.2, y > 0$. Find the covariance of x and Y.
2. Two random variables X and Y have the following joint p.d.f

   $f(x,y) = \begin{cases} 2-x-y ; 0 < x < 1 , 0 < y < 1 \\ 0 \quad ,elsewhere \end{cases}$. (i) Find the $var(x) and\ var\ (y)$

   (ii) The covariance between x and y. Also find ρ_{xy}.
3. If X and Y be discrete R.V's with p.d.f $f(x,y) = \frac{x+y}{21} , x = 1,2,3 ; y = 1,2.$
   (i) Find the mean and variance of X and Y, (ii) $cov(x,y)$ (iii) $r(x,y)$.

4. Two random variables X and Y have the following joint p.d.f

$$f(x,y) = \begin{cases} x+y\,;\ 0<x<1\,, 0<y<1 \\ 0 \qquad ,elsewhere \end{cases}$$. (i) Obtain the correlation co-efficient between X and y. (ii) Check whether X and Y are independent.

5. Find the coefficient of correlation between X and Y from the data given below.

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

6. Find the coefficient of correlation between industrial production and export using the following data.

Production (x):	55	56	58	59	60	60	62
Export (y) :	35	38	37	39	44	43	44

<u>SECOND HALF (All are 8- marks)</u>

<u>*I- Problems on regression line*</u>

1. Two random variables X and Y are related as $y = 4x + 9$. find the correlation coefficient between X and Y.

2. The two lines of regression are $8x - 10y + 66\,;\ 40x - 18y - 214 = 0$. The variance of X is 9. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y and find the variance of Y.

3. The two lines of regression are $8x - 10y + 66$ *and* $40x - 18y - 214 = 0$. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y.

4. The regression equations are $3x + 2y = 26$ *and* $6x + y = 31$. Find the correlation coefficient between x and y.

5. The equation of two regression lines are $3x + 12y = 19$ *and* $3x + 9y = 46$. Find $\bar{x}$, $\bar{y}$ and the correlation coefficient between x and y.

II- Transformation of random variables

1. If X and Y are independent random variables with p.d.f $e^{-x}, x \geq 0$; $e^{-y}, y \geq 0$ respectively. Find the density function of $U = \frac{X}{X+Y}$ $and\ V = X + Y$.

   Are U &V independent.

2. The joint P.d.f of X and Y is given by $f(x,y) = e^{-(x+y)}, x \geq 0, y \geq 0$.

   Find the probability density function of $U = \frac{X+Y}{2}$

3. If X and Y are independent exponential distributions with parameter 1 then find the P.d.f of $U = X - Y$.

4. Let (X, y) be a two-dimensional non-negative continuous random variable having the joint density $f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}, & x \geq 0, y \geq 0 \\ 0 & , elsewhere \end{cases}$.

   find the density of $U = \sqrt{x^2 + y^2}$

5. If the p.d.f of a two dimensional R.V (x,y) is given by $f(x,y) = x + y$,$0 \leq (x,y) \leq 1$. Find the p.d.f of $U = XY$.

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UNIT-3 RANDOM PROCESSES

PART – A

1. Define a stationary process (or)strictly stationary process (or)strict sense stationary process.
2. Consider the random process $X(t) = \cos(\omega_0 t + \theta)$, where θ is uniformly distributed in the interval $-\pi$ to π . Check whether X(t) is stationary or not?
3. When is a random process said to be ergodic.
4. Consider the Markov chain with tpm: $\begin{pmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is it irreducible? If not find the class. Find the nature of the states.
5. Define Markov chain and one-step transition probability.
6. State Chapman- Kolmogorow theorem.
7. What is a Markov process?
8. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the limiting distribution of the chain.
9. State any two properties of a Poisson process.
10. Prove that the difference of two independent poisson processes is not a poisson process.
11. If patients arrive at a clinic according to poisson process with mean rate of 2 per minute. Find the probability that during a 1-minute interval , no patients arrives.
12. The probability that a person is suffering from cancer is 0.001. Find the probability that out of 4000 persons (a) Exactly 4 suffer because of cancer, (b) more than 3 persons will suffer from the disease.
13. Define Stationary process.
14. Define Strictly stationary processes [sss]
15. Define Wide sense stationary processes [wss]
16. Define Ergodic random process.
17. What is meant by one-step transition probability?

PART – B

FIRST HALF (All are 8- marks)

I- To find the wide sense stationary process

1. If $(t) = \sin(\omega t + y)$, where 'y' is uniformly distributed in $(0,2\pi)$.
   Show that $x(t)$ is wide sense stationary process.
2. Show that the random process $x(t) = A\, cos(\omega t + \theta)$ is wide sense stationary, where $A\ and\ \omega$ are constants and θ is a uniformly distributed random in $(0,2\pi)$.
3. Show that the process $x(t) = Aco\,s\,\lambda t + Bsin\,\lambda t$ is wide sense stationary, where A and B are random variable if $E(A) = E(B) = 0\,, E(A^2) = E(B^2)$ and $E(AB) = 0$.
   (or)
   If $x(t) = Aco\,s\,\lambda t + Bsin\,\lambda t\,; t \geq 0$ is a random process where A and B are independent $N(0, \sigma^2)$ random variables examine the stationary of $x(t)$.
4. Let two random processes$\{x(t)\}\ and\ \{y(t)\}$ be defined as
   $x(t) = Aco\,s\,\omega t + Bsin\,\omega t, y(t) = Bco\,s\,\omega t - Asin\,\omega t$,where A and B are random variables and ω is a constant. If $E(A) = E(B) = 0\,, E(A^2) = E(B^2)$ and $E(AB) = 0$.Prove that $\{x(t)\}\ and\ \{y(t)\}$ is jointly wide sense stationary.

5. If $x(t) = y\,co\,s\,t + z\,sin\,t, \forall\,t$ where y and z are independent binary random variables, each of which assumes the values -1 and 2 with probabilities $\frac{2}{3}\ and\ \frac{1}{3}$ respectively. Prove that $\{x(t)\}$ is wide sense stationary.
6. Show that the random process $x(t) = A\,co\,s\,t + B\,sin\,t\,, -\infty < t < \infty$ is a WSS process, where A and B are independent random variables each of which has -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$.
7. The process $\{x(t)\}$ whose probability distribution under certain condition is given by

$$P\{x(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,3,.. \\ \frac{at}{1+at} \quad ,n = 0 \end{cases}$$

   Show that it is not stationary.

8. Let the random process be $x(t) = \cos(t + \varphi)$ where φ is a random variable with density function $f(\varphi) = \frac{1}{\pi}\ ,-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ check whether the process is stationary or not.
9. Examine whether the Poisson process $\{x(t)\}$ given by the probability law $P\{x(t)\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}\ , n = 0,1,2, \dots \dots \dots ....$ is not stationary.

V.PRAKASH,MS.c,M.Phil,B.Ed,
Associate Professor
Department Of Mathematics

II- Poisson process

1. Suppose that customer arrive at a bank according to a Poisson process with a mean rate 3 per minute; find the probability that during a time interval of 2 min (i) exactly 4 customers arrive ,(ii) more than 4 customers arrive and (iii) fewer than 4 customer in 2 minute interval.
2. Queries presented in a computer data base are following a Poisson process of rate $\lambda = 6$ queries per minute. An experiment consists of monitoring the data base for 'm' minutes and recording N (m) the number of queries presented. What is the probability that (i) no queries arrive in one minute interval, (ii) exactly 6 queries arriving in one minute interval and (iii) less than 3 queries arriving in a half minute interval.
3. On the average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is a Poisson variate, find the probability of sighting. (i) 6 ships in the next half-an-hour (ii) 4 ships in the next 2 hour (iii) at least 1 ship in the next 15 min (iv) at least 2 ship in the next 20 min
4. If particles are emitted from a radioactive source at the rate of 20 per hour, find the probability that exactly 5 particles are emitted during a 15 minute period.
5. A radioactive source emits particles at a rate 5 per minutes in accordance with poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-min period.

SECOND HALF (All are 8- marks)

III- Discrete parameter Markov process (Markov chain)

(a) To find the steady state distribution of the chain

1. An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals followed a highly distorted signal with no recognizable signal, whereas 20 out of 23 recognizable signals follow recognizable signals with no highly distorted signals between. Given that only highly distorted signals are not recognizable. Find the fraction of signals that are highly distorted.

2. A salesman's territory consists of three cities A,B and C, He never sells in the same city on successive days. If he sells in A, then the next day he sells in the city B. however, if he sells in either B or C, then the next day he twice as likely to sell in city A as in the other city. In the long run, how often does he sell in each of the cities.

(b) To find the probability distribution based on the initial distribution

1. The initial process of the Markov chain transition probability matrix is given by $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$ with initial probability $P_1^{(0)} = 0.4, P_2^{(0)} = 0.3, P_3^{(0)} = 0.3,$ Find $P_1^{(1)}, P_2^{(1)}, P_3^{(1)}$.

2. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drive one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drive to work if and only if a 6- appeared.
   Find (i) the probability that the drives to work in the long run.
   (ii) the probability that he takes a train on the 3rd day.

3. The transition probability matrix of the Markov chain $\{X_n\}$, n=1,2,3,…..having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial is $P^{(0)} = [0.7 \quad 0.2 \quad 0.1]$
   Find (i) $P(X_2 = 3)$ (ii) $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$

4. The tpm of a Markov chain with three states 0,1,2 is $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = {}^1/_3$, $i = 0,1,2$.
   Find (i) $P(X_2 = 2)$
   (iii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$.

5. The tpm of a Markov chain $\{X_n, n > 0\}$ have three states 0,1,2 $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ with initial probability $P^{(0)} = [0.5 \quad 0.3 \quad 0.2]$
   Find (i) $P(X_2 = 1)$ (ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$.

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QUESTION BANK

PERIOD: AUG – DEC-2022 **BATCH**: 2021-2025

BRANCH: ECE **YEAR/SEM:** II/03

SUB CODE/NAME: MA3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

UNIT 4 – VECTOR SPACES

PART – A

1. Define vector space
2. Define subspace
3. Define span[s]
4. Define linear combinations
5. Define linearly independent and linearly dependent
6. Define basis
7. Write down the standard basis for F^n.
8. Is {(1,4, −6), (1,5,8), (2,1,1), (0,1,0)} is a linearly independent subset of R^3 ?
9. Write the vectors w = (1, −2,5) as a linear combination of the vectors $v_1 = (1,1,1),\ v_2 = (1,2,3)\ and\ v_3 = (2,-1,1)$
10. Determine whether $w = (4, -7,3)$ can be written as a linear combination of $v_1 = (1,2,0)$ and $v_2 = (3,1,1)$ in R^3
11. For which value of k will the vector $u = (1, -2, k)$ in R^3 be a linear combination of the vectors $v = (3,0, -2)$ and $w = (2, -1,5)$?
12. Determine whether the set $W_1 = \{(a_1, a_2, a_3) \in R^3 : a_1 = a_3 + 2\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on R^3
13. Point out whether the set $W_1 = \{(a_1, a_2, a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$ is a subspace of R^3 under the operations of addition and scalar multiplication defined on $R3$
14. Point out whether $w = (3,4,1)$ can be written as a linear combination of $v_1 = (1, -2,1)$ and $v_2 = (-2, -1,1)$ in
15. Show that the vectors {(1,1,0), (1,0,1) and (0,1,1)} genarate F^3

16. Determine which of the following sets are basis for R^3

$(i)\quad \{(1,0,-1),(2,5,1),(0,-4,3)\}\quad (ii)\quad \{(1,-3,-2),(-3,1,3),(-2,-10,-2)\}$

17. Determine which of the following sets are basis for $P_2(R)$

(i) $\{-1-x+2x^2,\ 2+x-2x^2,\ 1-2x+4x^2\}$

(ii) $\{-1+2x+4x^2,\ 3-4x-210,\ -2-5x-6x^2\}$

18. Evaluate which of the following sets are bases for R^3 :

(i)$\{(1,0,-1), (2,5,1), (0,-4,3)\}$ (ii)$\{(-1,3,1), (2,-4,-3), (-3,8,2)\}$

PART – B

FIRST HALF (All are 8- marks)

I- Vector Spaces And Sub-Spaces

1. In any vector space V, the following statements are true,

(i) $0x=0, for\ each\ x\in v$

(ii) $a0=0, \forall a\in v$

(iii) $(-a)x=-(ax), \forall a\in F, \forall x\in v$

(iv) if $a\neq 0, then\ ax=0\Rightarrow 0$

2. Let V be the set of all polynomials of degree less than or equal to 'n' with real coefficients. Show that V is a vector space over$R$ with respect to polynomial addition and usual multiplication of real numbers with a polynomial

(or)

Prove that for $n>0$, the set P_n of polynomials of degree at most n consists of all polynomials of the from $a_0+a_1x+a_2x^2+\cdots \ldots\ldots\ldots.+a_nx^n$ is a vector space.

3. Let V denote the set of ordered pairs of real numbers. If (a_1,a_2) and (b_1,b_2) are elements of V and $c\in R, define\ (a_1,a_2)+(b_1,b_2)=(a_1+b_1,a_2+b_2)\ and\ c(a_1,a_2)=(ca_1,ca_2)$. Is V a vector space over R with these operations? Justify your answer.

4. Prove that any intersection of subspaces of a vector space V is a subspaces of V.

5. The intersection of the subspaces $w_1\ and\ w_2$ of the vector space V is also a subspace.

6. Prove that the span of any subset S of a vector space V is a subspace of V. moreover, any subspace of V that contains S must also contain the span of S.

(or)

The linear span L(S) of any subset of a vector space V(F) is a subspace of V(F). moreover $L(S) \subset W$.

7. Let W_1 and W_2 be subspaces of vector space V. prove that $w_1 \cup w_2$ is a subspace of V if and only if $w_1 \subseteq w_2\ (or)\ w_2 \subseteq w_1$

(or)

Let w_1 and w_2 be sub-spaces of vector space V, prove that $w_1 \cup w_2$ is a sub-space of V, iff one is contained in the other.

8. Show that the set $w = \{(a_1, a_2, a_3) \in R^3 : 2a_1 - 7a_2 + a_3 = 0\}$ is a sub-space of V

9. Show that the set $w = \{(a_1, a_2, a_3) \in R^3 : a_1 + 2a_2 - 3a_3 = 0\}$ is a sub-space of V

10. Prove that $w_1 = \{(a_1, a_2, \dots \dots \dots a_n) \in F^n;\ a_1 + a_2 + \cdots \dots \dots \dots + a_n = 0\}$ is a subspace of F^n, but $w_2 = \{(a_1, a_2, \dots \dots \dots a_n) \in F^n;\ a_1 + a_2 + \cdots \dots \dots \dots + a_n = 1\}$ is not a subspace .

11. Prove that of W is a subspace of a vector space V and $w_1, w_2, \dots \dots \dots, w_n$ are in W, then $a_1 w_1 + a_2 w_2 + \cdots \dots \dots \dots + a_n w_n \in W$ for any scalars $a_1, a_2, \dots \dots \dots a_n$

12. Show that W is in the subspace of R^4 spanned by v_1, v_2, v_3, where $w = \begin{bmatrix} 9 \\ -4 \\ -4 \\ 7 \end{bmatrix}$,

$$v_1 = \begin{bmatrix} 8 \\ -4 \\ -3 \\ 9 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ 3 \\ -2 \\ -8 \end{bmatrix}, v_3 = \begin{bmatrix} -7 \\ 6 \\ -18 \end{bmatrix}$$

13. If S and T are subsets of vector space V(F) ,then prove that
(i) $S \subset T \Rightarrow L(s) \subset L(T)$ (ii) $L(S \cup T) \Rightarrow L(S) + L(T)$ (iii) $L[L(S)] = L(S)$

SECOND HALF (All are 8- marks)

II-Linear Independent, Linear Dependent and basis

1. For each of the following list of vectors in R^3.Determine whether the first vector can be expressed as a linear combination of the other two (i) (-2,0,3),(1,3,0),(2,4,-1)
(ii) (3,4,1),(1,-2,1),(-2,-1,1).

2. For each of the following list of $P_3(R)$.Determine whether the first vector can be expressed as a linear combination of the other two

(i) $x^3 - 3x + 5, x^3 + 2x^2 - x + 1, x^3 + 3x^2 - 1$

(ii) $x^3 - 8x^2 + 4x , x^3 - 2x^2 + 3x + 1, x^3 - 2x + 3$

(iii) $4x^3 + 2x^2 - 6 , x^3 - 2x^2 + 4x + 1, 3x^3 - 6x^2 + x + 4$

3. Determine whether the following sets are linearly independent (or) linearly dependent.

(i) $\left\{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ -4 & 4 \end{bmatrix}\right\} in\ M_{2x2}(R)$

(ii) $\left\{\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix}\right\} in\ M_{2x2}(R)$

(iii) $\left\{\begin{bmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{bmatrix}, \begin{bmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{bmatrix}, \begin{bmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{bmatrix}\right\} in\ M_{2x3}(R)$

4. Determine the following sets are linearly independent (or) linearly dependent

(i) $\{x^3 + 2x^2 , -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\} in\ \ P_3(R)$

(ii) $\{x^3 - x , 2x^2 + 4 , -2x^3 + 3x^2 + 2x + 6\} in\ \ P_3(R)$

(iii) $\{(1, -1,2), (1, -2,1), (1,1,4)\}\ in\ R^3$

(iv) $\{(1, -1,2), (2,0,1), (-1,2, -1)\}\ in\ R^3$

5. Prove that vectors $u_1 = (2, -3,1)$, $u_2 = (1,4, -2)$, $u_3 = (-8,12, -4)$, $u_4 = (1,37, -17), u_5 = (-3, -5,8)$ Generate R^3. Find a subset of the set $\{u_1, u_2, u_3, u_4\}$ that is a basis for R_3.

6. In each part ,determine whether the given vector is in the span S

(i) $(-1,2,1), S = \{(1,0,2), (-1,1,1)\}$

(ii) $(-1,1,1,2), S = \{(1,0,1, -1), (0,1,1,1)\}$

(iii) $-x^3 + 2x^2 + 3x + 3,\ S = \{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\}$

V.PRAKASH,MS.c,M.Phil,B.Ed,
Associate Professor
Department Of Mathematics

7. Let $S = \{(1,1,0), (1,0,1), (0,1,1)\}$ be a subset of the vector space F^3.prove that if F=R, then S is linearly independent.

8. Show that the set $\{1, x, x^2, \dots \dots \dots \dots . x^n\}$ is a linearly independent in $P_n(F)$.

9. Let V be a vector space, and let $S_1 \subseteq S_2 \subseteq V$. If S_1 is linearly dependent, then S_2 is linearly dependent.

10. Let S be a linearly independent subset of a vector space V, and V be a vector in v that is not in S. then $S \cup \{V\}$ is linearly dependent iff $V \in span\ (S)$.

11. Let V be a vector space over a field of characteristic not equal to zero, let U and V be distinct vectors in V. Prove that $\{u, v\}$ is linearly independent iff $\{u + v, u - v\}$ is linearly independent.

12. Let V be a vector space over a field of characteristic not equal to zero, let U ,V and W be distinct vectors in V. Prove that $\{u, v, w\}$ is linearly independent iff $\{u + v, v + w, u + w\}$ is linearly independent.

13. Let U,V and W be distinct vectors of a vector space V. Show that if $\{u, v, w\}$ is a basis for V, then $\{u + v + w, v + w, u\}$ is also basis for V.

14. If a vector space V is generated by a finite set S, then some subset of S is a basis for V. Hence V has a finite basis.

15. Let V be a vector space and $B = \{u_1, u_2, \dots \dots ., u_n\}$ be a subset of V, then B is a basis for V. If each $v \in V$ can be uniquely expressed as a linear combination of vector of B. (i.e) It can be expressed in the form $V = a_1u_1 + a_2u_2 + \cdots \dots . + a_nu_n$ for unique scalar $a_1, a_2, \dots \dots ., a_n$.

16. If W_1, W_2 are two subspaces of a finite dimensional vector space V then $\dim(W_1 + W_2) = \dim W_1 + dimW_2 - \dim(W_1 \cap W_2)$ and hence deduce that if $V = W_1 + W_2$, then $\dim(V) = \dim W_1 + dimW_2$.

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UNIT 5 – LINEAR TRANSFORMATION AND INNER PRODUCT SPACES

PART – A

1. If $T: V \to W$ be a linear transformation then prove that $T(x - y) = x - y$ for all $x, y \in V$
2. Prove that the transformation T is linear if and only if $T(cx + y) = cT(x) + T(y)$
3. Illustrate that the transformation $T: R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_2)$ is linear
4. Evaluate that the transformation $T: R^3 \to R^2$ defined by by
   T $(a_1, a_2, a_3) = (a_1 - a_2, a_1 - a_3)$ is linear.
5. Describe explicitly the linear transformation $T: R^2 \to R^2$such that $T(2,3) = (4,5) and$
   $T(1,0) = (0,0)$
6. Illustrate that the transformation $T: R^2 \to R^3$defined by $T(x, y) = (x + 1, 2y, x + y)$
   is not linear
7. Is there a linear transformation $T: R^3 \to R^3$such that $T(1,0,3) = (1,1)$ and
   $(-2,0,-6) = (2,1)$?
8. Find the matrix [T]e whose linear operator $is\ T(x, y) = (5x + y, 3x - 2y)$
9. Find the matrix representation of T whose basis is $f1 = (1,2)$ $f2 = (2,3)$ such that
   $(x, y) = (2y, 3x - y)$
10. Define eigen value and eigen vector of linear operator T.
11. State Cayley-Hamilton Theorem
12. Find the matrix A whose minimum polynomial is $t^3 - 5t^2 + 6t + 8$
13. Suppose λ is an eigen value of an invertible operator T.
    Show that λ^{-1} is an eigen value of ${}^{-1}$.
14. Define norm
15. Define orthogonal
16. Define orthonormal

17. Define orthogonal complement.

18. Define adjoint operator.

19. Let $x = (2,1 + i, i) and\ y = (2 - i, 2,1 + 2i)$ be a vector in C^3 compute $< x, y >$.

20. Find the norm and distance between the vectors u=(1,0,1) and v=(-1,1,0)

21. Find the norm of the vector u=(1,-1,1) and v=(-1,1,0) in R^3 with respect to the inner product defined by $< u, v > = u_1v_1 + 2u_2v_2 + 3u_3v_3$.

22. Find the norm of v = (3,4) ∈ R^2 with respect to the usual product.

23. Consider $f(t) = 3t - 5\ and\ g(t) = t^2$ in the polynomial space p(t) with inner product $< f, g > = \int_0^1 f(t0g(t)dt, then\ find\ \|f\|\ and\ \|g\|$.

24. Prove that in an inner product space V, for any $u, v \in V. \|u + v\|^2 + \|u + v\|^2 = 2\|u\|^2 + 2\|u\|^2$

25. In ([0,1]), let $f(t) = t$, $g(t) = e^t$ Evaluate $< f, g >$.

26. Let R2 and $S = \{(1,0), (0,1)\}$. Check whether S is orthonormal basis or not.

27. Let $S = \left\{\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(\frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)\right\}$ .verify S is orthonormal basis or not.

28. State Cauchy Schwarz inequality and Triangle inequality.

PART – B

FIRST HALF (All are 8- marks)

1. Let V and W be the vector spaces and $T: V \to W$ be linear. Then prove that N(T) and R(T) are subspaces of V and W respectively.

2. Let V and W be vector spaces and let $T: V \to W$ be a linear transformation. If β ={$v_1, v_2, \ldots, v_n$}is a basis for V,then show that $SpanT(\beta) = R(T)$. Also prove that T is one –to-one if and only if $N(T)$ = {0}

3. Dimension theorem: Let V and W be the vector spaces and $T: V \to W$ be linear. If V is finite-dimensional then nullity(T)+rank(T)=dim(V).

4. Let V and W be the vector spaces and $T: V \to W$ be linear, then T is one-to-one iff N(T)={0}.

5. Let $T: R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find the basis for N(T) and the nullity of T.

6. Let $T: R^2 \to R^3$ defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$. Find the basis for and compute N(T).

7. Let $T: R^3 \to R^2$ defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find the basis forR(T) and compute R(T).

8. Let $T: R^2 \to R^3$ defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$. Find the basis for and compute N(T).

9. Let $T: R^3 \to R^2$,defined by $T(x, y, z) = (x + y, y + z)$ then $B_1 = \{(-1,1,1), (1,-1,1), (1,1,-1)\}$ and $\mathrm{B}_2 = \{(1,0), (0,1)\}$ then find $[T]_{B_1}^{B_2}$

10. Let $T: R^2 \to R^3$,defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ then and $\mathrm{B}_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,0,0), (0,1,0), (0,0,1)\}$.

11. Let $T: R^3 \to R^3 and\ U: R^3 \to R^3$ be the linear transformation respectively defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$. Then prove that $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$

12. Let T be the linear operator on R3 defined by $T\begin{pmatrix}\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}\end{pmatrix} = \begin{bmatrix}4a_1 + a_3\\2a_1 + 3a_2 + 2a_3\\a_1 + 4a_3\end{bmatrix}$. Determine the Eigenspace of T corresponding to each Eigenvalue. Let B be the standard ordered basis for R^3.

13. Let T be a linear operator on $P_2(R)$ defined by $T[f(x)] = f(1) + f'(0)x + [f'(0) + f''(0)]x^2$. Test for diagonalisability.

14. Let T be a linear operator on a vector space V, and let $\lambda_1, \lambda_2, \lambda_3 \dots \dots \dots \lambda_k$ be distinct eigenvalues of T. For each $i = 1,2,3, \dots . k$, let Si be a finite linearly independent subset of the Eigen space $E_{\lambda i}$. Then $S = S_1 \cup S_2 \cup \dots \dots \cup S_k$ is a linearly independent subset of V.

15. Let T: $P_2(R) \to P_3(R)$be defined by $T[f(x)] = xf(x) + f'(x)$is linear. Find the bases for both (T), (T), nullity of T, rank of T and determine whether T is one –to-one or onto.

16. Let T: $R^3 \to R^3$be a linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Evaluate a basis and dimension of null space N(T) and range space R(T) and range space R(T). Also verify dimension theorem.

17. Let V and W be vector spaces over F,and suppose that$\{v_1, v_2 \dots \dots \dots . v_n\}$ is a basis for V, For $w_1, w_2 . w_n$ in W Prove that there exists exactly one linear transformation $T: V \to W$ such that $T(v_i) = w_i$ for i=1,2,…n

18. Suppose that T is one –to-one and that s is a subset of V .Prove that S is linearly independent if and only if T(S) is linearly independent .
Suppose β= $\{v1, v2., \dots \dots \dots vn\}$ is abasis for V and T is one –to-one and onto
Prove that $T(\beta)$= $\{T(v_1), T(v_2), T(v_n)\}$ is a basis for W

V.PRAKASH,MS.c,M.Phil,B.Ed,
Associate Professor
Department Of Mathematics

19. Let V and W be vector spaces with subspaces V_1 W_1respectively. If T: $\rightarrow W$ is linear .Prove that T(V_1) is a subspace of w and that x∈ V: $T(x) \in W_1$}is a subspace of V.

20. If T: $R^4 \rightarrow R^3$ is a linear transformation defined T{x_1, x_2, , x_3, x_4}=($x_1 - x_2 + x_3 + x_4$, $_1 + 2x_3 - x_4$, $_1 + x_2 - 3x_3 - 3x_4$) for {x_1, x_2, , x_3, x_4} ∈ R then verify Rank(T)+Nullity(T)=dim R^4 find yhe bases of N(T) and R(T)

21. For a linear operator T: $R^3 \rightarrow R^3$ defined as $T(a, b, c) = (-7a - 4b + 10c, 4a - 3b + 8c, -2a + b - 2c)$,Point out the eigen values of T and an ordered basis $\beta$for R^3 such that the matrix of the given transformation with the respect to the new resultant basis β is a diagonal matrix

22. Let T be a linear operator $(a, b, c) = (-4a + 3b - 6c, 6a - 7b + 12c, 6a - 6b + 11c)$, be the ordered basis then find $[T]$ which is a diagonal matrix.

<u>SECOND HALF (All are 8- marks)</u>

1. let $V = M_{mxn}(F) and\ define < A, B >= tr\,(B^*A)$ for $A, B \in V$,the trace of a matrix A is defined by $tr(A) = \sum_{i=1}^{k} A_{ii}$. Verify <. , .> is an inner product space.

2. Let V be a real (or) complex vector space and let B be a basis for V for $x, y \in V$ there exists $v_1, v_2, \ldots \ldots \ldots . . v_n \in B\ such\ that\ x = \sum_{i=1}^{n} a_i v_i\ and\ y = \sum_{i=1}^{n} b_i v_i$. Define $< x, y > = \sum_{i=1}^{n} a_i \overline{b_i}$ .prove that <. , .> is an inner product on V and that B is an orthonormal basis V.

3. Let $x = (2,1 + i, i\,) and\ (2 - i, 2,1 + 2i)$ be vectors in C^3.compute (i) $< x, y >$ (ii) $\|x\|$ (iii) $\|y\|$ (iv) $\|x + y\|$ (v) Cauchy's inequality (vi) Triangle inequatily.

4. Let V be an inner product space. Then for $x, y, z \in V\ andc \in F$,the following statements are true.

i) $< x, y + z >=< x, y > + < x, z >$
ii) $< x, cy >= \bar{c} < x, y >$
iii) $< x, 0 >=< 0, x >= 0$
iv) $< x, x >=< x, z >\ \ \forall x \in v, then\ y = z$

5. Let V be an inner product space over F, then for all $, y \in V\ and\ c \in F$, the following statements are true

i) $\|cx\| = |c|\|x\|$
ii) $\|x\| = 0, iff\ x = 0$
iii) $|< x, y >| \leq \|x\|\|y\|$
iv) $\|x = y\| \leq \|x\| + \|y\|$

6. State and prove Cauchy-Schwarz inequality and Triangle inequality in an inner product space.

7. Let V be an inner product space. Prove that
(a) $\|x \pm y\|2 = \|x\|2 \pm 2R < x, y > +\|y\|^2$ for all x, y ∈ V, where $R < x, y >$ denotes the real part of the complex number $< x, y >$.
(b) $|\|x\| - \|y\||^2 \leq \|x - y\|$ for all x, y ∈ V.

8. Let V be an inner product space over F. prove that polar identities for all $x, y \in V$.
$< x, y > = \frac{1}{4}\|x + y\|^2 - \frac{1}{4}\|x - y\|^2$ if F=R.

9. show that in R^3, the vectors (1,1,0), (1,-1,1), (-1,1,2) are orthogonal, are they orthonormal? Justify.

10. Let V be the vector space of polynomial with inner product given by
$< x, y > = \int_0^1 f(t)g(t)dt$. let $(t) = t + 2$ and $g(t) = t^2 - 2t - 3$,
find (i) $< f, g >$ (ii) $\|f\|$ (iii) $\|g\|$.

11. Let $\{v_1, v_2, \dots \dots \dots . . v_k\}$ be an orthogonal set in V and $a_1, a_2, \dots \dots \dots . . a_k$ be scalars, Prove that $\left\|\sum_{i=1}^{k} a_i v_i\right\|^2 = \sum_{i=1}^{k} |a_i|^2 \|v_i\|^2$.

12. Evaluate using the Gram Schmidt Process to the given subset
$S = \{(1,1,1), (0,1,1), (0,0,1)\}$ and $x = (1,0,1)$ of the inner product space $V = R^3$ to obtain (i) an orthogonal basis for span(S).
(ii) Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S)
(iii) compute the Fourier coefficients of the given vector relative to β.

13. Evaluate by the Gram Schmidt Process to the given subset
$S = \{(1, -2, -1, 3), (3, 6, 3, -1), (1, 4, 2, 8)\}$ and $x = (-1, 2, 1, 1)$ of the inner product space $= R^4$ to obtain (i) an orthogonal basis for span(S).
(ii)Then normalize the vectors in this basis to obtain an orthonormal basis β for span(S)
(iii)compute the Fourier coefficients of the given vector relative to β.

14. Apply the Gram-Schmidt process to the given subsets S of the inner product space V to obtain
i) orthogonal basis for span (s)
ii) Normalize the vectors in the basis to obtain an orthonormal basis for span(s)
Let $V = P_2(R)$ with the inner product $< f, g > = \int_0^1 f(t)g(t)dt,$ and $\{1, x, x^2\}$

15. Apply the Gram-Schmidt process to the given subsets
$S = \{(1, i, 0), (1 - i, 2, 4i)\}$ and $x = (3 + i, 4i, -4)$ of the inner product space V= R^3
i) To obtain an orthogonal basis for span (s)
ii) Normalize the vectors in the basis to obtain an orthonormal basis for span (s)
iii) Compute to Fourier coefficients of the given vector.

16. State and prove the Gram-Schmidt orthogonalization theorem (or)

Let V be an inner product space and $S = \{w_1, w_2, \dots \dots . w_n\}$ be a linearly independent subset of V. Define $S' = \{v_1, v_2, \dots \dots . v_n\}$ when $v_1 = w_1$ and $v_k = w_k - \sum_{j=1}^{k-1} \frac{<w_k, v_j>}{\|vj\|^2} vj$ for $2 \leq k \leq n$. Then S' is an orthogonal set of non-zero vectors such that Span (s') = Span (s).

17. Let V be an inner product space, $S \text{ and } S_0$ be the subset of V and W be a finite dimensional subspace of V. Prove that the following results

i) $S_0 \subseteq S \implies S^{\perp} \subseteq S_0^{\perp}$

ii) $S \subseteq (S^{\perp})^{\perp}$ $, so\ span\ (s) \subseteq (S^{\perp})^{\perp}$

iii) $W = (W^{\perp})^{\perp}$

iv) $V = W \oplus W^{\perp}$

18. Let V be a finite dimensional inner product space, and let T be a linear operator on V. Then there exists a unique function $T^*: V \to V$ such that $< T(x), y > = < x, T^*(y) > for\ all\ x, y \in V$ and T^* is linear.

19. Let V be an inner product space, T and U be linear operators on V.then

i) $(T + U)^* = T^* + U^*$

ii) $(CU)^* = \bar{C}T^*, for\ any\ c \in F$

iii) $(TU)^* = U^*T^*$

iv) $T^{**} = T$

v) $I^* = I$

20. $Let\ A\ and\ B\ be\ n\ X\ n\ matrices. Then\ prove\ that$

(i) $(A + B)* = A* + B*$

(ii) $(cA)* = \bar{c}A*\ for\ all\ c \in F$

(iii) $(AB)* = B * A*$

(iv) $A** = A$

(v) $I* = I$

21. Suppose that S=$\{v_1, v_2, \dots v_k\}$ is an orthonormal set in an n-dimensional inner product space V. Then Prove that

(i) S can be extended to an orthonormal basis $\{v_1, v_2, \dots v_k, v_{K+1} \dots \dots v_n\}$ for V

(ii) If W= span(S),then s_1=$\{v_{K+1}, v_{K+2} \dots \dots . v_n\}$ is an orthonormal basis for $w^{\perp}$

(iii) If W is any subspace of V , then dim(v)=dim(w)+dim($w^{\perp}$)